

A Quark-Meson Model for Heavy Mesons: Semi-Leptonic Decays

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Abstract

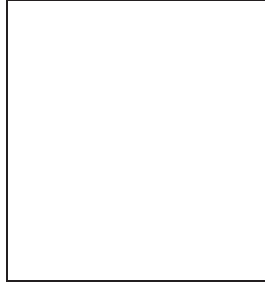
I consider a model for heavy meson decays based on an effective quark-meson Lagrangian. The model is constrained by the known symmetries of QCD in the $m_Q \rightarrow \infty$ limit for the heavy quarks, and chiral symmetry in the light quark sector. Using a limited number of free parameters it is possible to compute several phenomenological quantities, e.g. the leptonic B and B^{**} decay constants; the Isgur-Wise form factors: ξ , $\tau(3/2)$, $\tau(1/2)$, describing the semi-leptonic decays $B \rightarrow D^{(*)}l\nu$, $B \rightarrow D^{**}l\nu$; the form factors for heavy to light decays $B \rightarrow \rho l\nu$, $B \rightarrow a_1 l\nu$. I show that the semileptonic heavy-to-light form factors calculated in the model fulfill the general relations that hold in QCD in the large energy limit for the final hadron.

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A QUARK MODEL FOR HEAVY MESONS: SEMI-LEPTONIC DECAYS

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1 Introduction

The model I discuss in this talk conjugates the symmetry approach of effective Lagrangians for heavy and light mesons¹ together with dynamical assumptions on chiral symmetry breaking and confinement. The reason for adding dynamical assumption is on one side the higher predictivity one obtains (less free parameters with respect to the symmetry approach alone) and the possibility of testing the predictions stemming from these assumptions against experimental data.

The model is also suitable for the description of higher spin heavy mesons as they can be included in the formalism in a very easy way². On the contrary the inclusion of higher order corrections, requires the determination of new free parameters, which proliferate as new orders are added to the expansion. In this sense the model allows a simple and intuitive approach to heavy-meson processes if it is kept at lowest order, while it loses part of its predictive power if corrections have to be included.

1.1 Heavy meson field

The lowest negative parity spin doublet (P, P^*) (for charm for instance, they correspond to D and D^*) can be represented by a 4×4 Dirac matrix H , with one spinor index for the heavy quark and the other for the light degrees of freedom.

An explicit matrix representation is:

$$H = \frac{(1 + \not{v})}{2} [P_\mu^* \gamma^\mu - P \gamma_5] \quad (1)$$

$$\bar{H} = \gamma_0 H^\dagger \gamma_0 \quad (2)$$

Here v is the heavy meson velocity, $v^\mu P_{a\mu}^* = 0$ and $M_H = M_P = M_{P^*}$. Moreover $\not{v}H = -H\not{v} = H$, $\bar{H}\not{v} = -\not{v}\bar{H} = \bar{H}$ and $P^{*\mu}$ and P are annihilation operators normalized as follows:

$$\langle 0 | P | Q \bar{q} (0^-) \rangle = \sqrt{M_H} \quad (3)$$

$$\langle 0 | P^{*\mu} | Q \bar{q} (1^-) \rangle = \epsilon^\mu \sqrt{M_H} \quad (4)$$

The formalism for higher spin states was introduced by Falk and Luke³. In matrix notation, analogous to the one used for the negative parity states, they are described by

$$S = \frac{1 + \not{v}}{2} [P_{1\mu}^{*\prime} \gamma^\mu \gamma_5 - P_0] \quad (5)$$

and

$$T^\mu = \frac{1 + \not{v}}{2} \left[P_2^{*\mu\nu} \gamma_\nu - \sqrt{\frac{3}{2}} P_{1\nu}^* \gamma_5 \left(g^{\mu\nu} - \frac{1}{3} \gamma^\nu (\gamma^\mu - v^\mu) \right) \right] \quad (6)$$

1.2 Interaction Lagrangian

The part of the quark-meson effective Lagrangian involving heavy and light quarks and heavy mesons is :

$$\begin{aligned} \mathcal{L}_{h\ell} = & \bar{Q}_v i v \cdot \partial Q_v - \left(\bar{\chi} (\bar{H} + \bar{S} + i \bar{T}_\mu \frac{D^\mu}{\Lambda_\chi}) Q_v + h.c. \right) \\ & + \frac{1}{2G_3} \text{Tr}[(\bar{H} + \bar{S})(H - S)] + \frac{1}{2G_4} \text{Tr}[\bar{T}_\mu T^\mu] \end{aligned} \quad (7)$$

where Q_v is the effective heavy quark field, χ is the light quark field, G_3, G_4 are coupling constants and $\Lambda_\chi (= 1 \text{ GeV})$ has been introduced for dimensional reasons. Lagrangian (7) has heavy spin and flavour symmetry.

Note that the fields H and S have the same coupling constant. There is no symmetry reason for them to be the same. By putting these two coupling constant equal, one assumes that this effective quark-meson Lagrangian can be justified as a remnant of a four quark interaction of the NJL type by partial bosonization.

The cut-off prescription is also part of the dynamical information regarding QCD which is introduced in the model. In the infrared the model is not confining and its range of validity can not be extended below energies of the order of Λ_{QCD} . In practice one introduces an infrared cut-off μ , to take this into account.

Models related to the one discussed here, with different regularization prescriptions and slightly different approach are the one of Bardeen and Hill⁴ and Holdom and Sutherland⁵. The cut-off prescription used here is implemented via a proper time regularization. After continuation to the Euclidean space it reads, for the light quark propagator:

$$\int d^4 k_E \frac{1}{k_E^2 + m^2} \rightarrow \int d^4 k_E \int_{1/\mu^2}^{1/\Lambda^2} ds e^{-s(k_E^2 + m^2)} \quad (8)$$

Table 1: Form factors and slopes. Δ_H in GeV.

Δ_H	$\xi(1)$	ρ_{IW}^2	$\tau_{1/2}(1)$	$\rho_{1/2}^2$	$\tau_{3/2}(1)$	$\rho_{3/2}^2$
0.3	1	0.72	0.08	0.8	0.48	1.4
0.4	1	0.87	0.09	1.1	0.56	2.3
0.5	1	1.14	0.09	2.7	0.67	3.0

where μ and Λ are infrared and ultraviolet cut-offs.

The cut-off prescription is similar to the one used by Ebert et al.⁶, with $\Lambda = 1.25$ GeV; the numerical results are not strongly dependent on the value of Λ . The constituent mass m in the NJL models represents the order parameter discriminating between the phases of broken and unbroken chiral symmetry and can be fixed by solving a gap equation, which gives m as a function of the scale mass μ for given values of the other parameters. Here I take $m = 300$ MeV and $\mu = 300$ MeV.

2 Semi-leptonic Decays

In the following I describe only part of the results obtained using the quark-meson Lagrangian concerning semi-leptonic decays. More details concerning the leptonic decay constants and semi-leptonic decays for heavy to heavy mesons can be found in², semi-leptonic decays for heavy to light mesons in⁷, strong decay constants for higher multiplets in⁸. A discussion of the large energy limit of the semileptonic heavy-to-light form factors not included in previous publications is included at the end.

2.1 Heavy-to-Heavy Semi-leptonic Decays

As an example of the quantities that can be analytically calculated in the model, one can examine the Isgur-Wise function ξ :

$$\langle D(v') | \bar{c} \gamma_\mu (1 - \gamma_5) b | B(v) \rangle = \sqrt{M_B M_D} C_{cb} \xi(\omega) (v_\mu + v'_\mu) \quad (9)$$

where $\omega = v \cdot v'$ and C_{cb} is a coefficient containing logarithmic corrections depending on α_s ; within our approximation it can be put equal to 1: $C_{cb} = 1$. At leading order $\xi(1) = 1$. The same universal function ξ also parametrises $B \rightarrow D^*$ semileptonic decay.

One can compute in a similar way the form factors describing the semi-leptonic decays of a meson belonging to the fundamental negative parity multiplet H into the positive parity mesons in the S and T multiplets. Examples of these decays are $B \rightarrow D^{**} l \nu$ where D^{**} can be either a S state or a T state. These decays are described by two form factors $\tau_{1/2}, \tau_{3/2}$ ⁹ which can be computed by a loop calculation similar to the one used to obtain $\xi(\omega)$.

The numerical results are reported in Table 1. For a comparison with other calculations of these form factors see Morenas et al.¹⁰.

An important test of our approach is represented by the Bjorken sum rule, which states

$$\rho_{IW}^2 = \frac{1}{4} + \sum_k \left[|\tau_{1/2}^{(k)}(1)|^2 + 2|\tau_{3/2}^{(k)}(1)|^2 \right]. \quad (10)$$

Numerically we find that the first excited resonances, i.e. the S and T states ($k = 0$) practically saturate the sum rule for all the three values of Δ_H . From the sum rule one can also derive bounds for the slope ρ_{IW}^2 of the Isgur-Wise function. Neglecting order α_s and $1/m_Q$ corrections

	CQM ⁷	Potential Model ¹⁴	SR ¹⁵	Latt. + LCSR ¹⁶
$V^\rho(0)$	-0.01 ± 0.25	0.45 ± 0.11	0.6 ± 0.2	$0.35^{+0.06}_{-0.05}$
$A_1^\rho(0)$	0.58 ± 0.10	0.27 ± 0.06	0.5 ± 0.1	$0.27^{+0.05}_{-0.04}$
$A_2^\rho(0)$	0.66 ± 0.12	0.26 ± 0.05	0.4 ± 0.2	$0.26^{+0.05}_{-0.03}$
$A_0^\rho(0)$	0.33 ± 0.05	0.29 ± 0.09	0.24 ± 0.02	$0.30^{+0.06}_{-0.04}$

Table 2: Form factors for the transition $B \rightarrow \rho$ at $q^2 = 0$. The results of CQM are compared with the outcome of other theoretical calculations: potential models, QCD sum rules (SR), calculations involving both lattice and light cone sum rules. The large error of $V^\rho(0)$ in our approach is due to the large cancellation between the direct and polar contribution.

the lower bound (Bjorken bound¹¹) is $1/4$ while the upper bound (Voloshin bound¹²) is 0.75 . The Bjorken bound is satisfied by our result in table 1. The Voloshin bound is only marginally satisfied. However the Voloshin bound is less stringent as it depends on further assumptions¹³.

2.2 Heavy-to-Light Semi-leptonic Decays

The form factors for the semileptonic decays $B \rightarrow \rho \ell \nu$ can be written as follows ($q = p - p'$):

$$\begin{aligned}
\langle \rho^+(\epsilon(\lambda), p') \mid \bar{u} \gamma_\mu (1 - \gamma_5) b \mid \bar{B}^0(p) \rangle = & \frac{2V(q^2)}{m_B + m_\rho} \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p^\alpha p'^\beta \\
& - i \epsilon_\mu^* (m_B + m_\rho) A_1(q^2) + i (\epsilon^* \cdot q) \frac{(p + p')_\mu}{m_B + m_\rho} A_2(q^2) \\
& + i (\epsilon^* \cdot q) \frac{2m_\rho}{q^2} q_\mu [A_3(q^2) - A_0(q^2)] ,
\end{aligned} \tag{11}$$

where

$$A_3(q^2) = \frac{m_B + m_\rho}{2m_\rho} A_1(q^2) - \frac{m_B - m_\rho}{2m_\rho} A_2(q^2) , \tag{12}$$

The calculation of these form factors in the model arises from two sources: “direct” diagrams in which the weak current couples directly to the quarks in the light and heavy mesons, and “polar” contributions where the weak current is coupled to the heavy and light mesons by an intermediate meson state. The expressions for the form factors are quite lengthy and can be found in⁷. The numerical results are in Table 2. For the $B \rightarrow \rho \ell \nu$ decay width and branching ratio the model predicts (using $V_{ub} = 0.0032$, $\tau_B = 1.56 \times 10^{-12}$ s):

$$\begin{aligned}
\mathcal{B}(\bar{B}^0 \rightarrow \rho^+ \ell \nu) &= (2.5 \pm 0.8) \times 10^{-4} \\
\Gamma_0(\bar{B}^0 \rightarrow \rho^+ \ell \nu) &= (4.4 \pm 1.3) \times 10^7 \text{ s}^{-1} \\
\Gamma_+(\bar{B}^0 \rightarrow \rho^+ \ell \nu) &= (7.1 \pm 4.5) \times 10^7 \text{ s}^{-1} \\
\Gamma_-(\bar{B}^0 \rightarrow \rho^+ \ell \nu) &= (5.5 \pm 3.7) \times 10^7 \text{ s}^{-1} \\
(\Gamma_+ + \Gamma_-)(\bar{B}^0 \rightarrow \rho^+ \ell \nu) &= (1.26 \pm 0.38) \times 10^8 \text{ s}^{-1}
\end{aligned} \tag{13}$$

where $\Gamma_0, \Gamma_+, \Gamma_-$ refer to the ρ helicities. This decay was observed by the CLEO collaboration¹⁷:

$$\mathcal{B}(B^0 \rightarrow \rho^- \ell^+ \nu) = (2.5 \pm 0.4^{+0.5}_{-0.7} \pm 0.5) \times 10^{-4} . \tag{14}$$

in good agreement with what is predicted by the constituent quark model.

2.3 Heavy-to-Light Form Factors and Final Hadron Large Energy Limit

It is interesting to examine a particular limit for the $B \rightarrow \rho$ semileptonic form factors, namely the one of heavy mass for the initial meson and of large energy for the final one. In this limit

the expressions of the form factors simplify and for $B \rightarrow V l \nu$, where V is the ρ in the following example, they reduce only to two independent functions (see J. Charles et al.¹⁸ for details). The four-momentum of the heavy meson is written as $p = M_H v$ in terms of the mass and the velocity of the heavy meson. The four-momentum of the ρ is written as $p' = E n$ where $E = v \cdot p'$ is the energy of the light meson and n is a four-vector defined by $v \cdot n = 1, n^2 = 0$. This peculiar large energy limit is defined as :

$$\Lambda_{QCD, m_V} < M_H, E \quad (15)$$

keeping v and n fixed and m_V is in our example the mass of the ρ . In agreement with J. Charles et al.¹⁸ I find the following result :

$$A_0(q^2) = \left(1 - \frac{m_V^2}{M_H E}\right) \zeta_{||}(M_H, E) + \frac{m_V}{M_H} \zeta_{\perp}(M_H, E) \quad (16)$$

$$A_1(q^2) = \frac{2E}{M_H + m_V} \zeta_{\perp}(M_H, E) \quad (17)$$

$$A_2(q^2) = \left(1 + \frac{m_V}{M_H}\right) \left[\zeta_{\perp}(M_H, E) - \frac{m_V}{E} \zeta_{||}(M_H, E) \right] \quad (18)$$

$$V(q^2) = \left(1 + \frac{m_V}{M_H}\right) \zeta_{\perp}(M_H, E). \quad (19)$$

From (19) and (17) one can immediately obtain the relation:

$$\frac{V(q^2)}{A_1(q^2)} = \frac{(M_H + m_V)^2}{M_H^2 + m_V^2 - q^2} \quad (20)$$

The explicit expressions for $\zeta_{||}$ and ζ_{\perp} are as follows in the constituent quark model:

$$\begin{aligned} \zeta_{||}(M_H, E) &= \frac{\sqrt{M_H Z_H} m_V^2}{2E f_V} \left[I_3\left(\frac{m_V}{2}\right) - I_3\left(-\frac{m_V}{2}\right) \right. \\ &\quad \left. + 4\Delta_H m_V Z(\Delta_H) \right] \sim \frac{\sqrt{M_H}}{E} \end{aligned} \quad (21)$$

$$\zeta_{\perp}(M_H, E) = \frac{\sqrt{M_H Z_H} m_V^2}{2E f_V} \left[I_3(\Delta_H) + m_V^2 Z(\Delta_H) \right] \sim \frac{\sqrt{M_H}}{E} \quad (22)$$

where terms proportional to the constituent light quark mass m have been neglected. Z_H is the renormalization constant for the heavy meson². Δ_H is the difference between the heavy meson and the heavy quark mass. This quantity stays finite in the limit. The full expressions for the form factors also involve $\Delta' = \Delta_H - E$. However using the asymptotic expansion for the error function entering in the integrals one can show that in the limit $E \rightarrow \infty$ these terms vanish. The functions appearing in the previous expression are:

$$\begin{aligned} I_3(\Delta) &= -\frac{iN_c}{16\pi^4} \int^{\text{reg}} \frac{d^4 k}{(k^2 - m^2)(v \cdot k + \Delta + i\epsilon)} \\ &= \frac{N_c}{16\pi^{3/2}} \int_{1/\Lambda^2}^{1/\mu^2} \frac{ds}{s^{3/2}} e^{-s(m^2 - \Delta^2)} (1 + \text{erf}(\Delta\sqrt{s})) , \end{aligned} \quad (23)$$

where erf is the error function and

$$Z(\Delta) = \frac{iN_c}{16\pi^4} \int^{\text{reg}} \frac{d^4 k}{(k^2 - m^2)[(k + q)^2 - m^2](v \cdot k + \Delta + i\epsilon)} . \quad (24)$$

The fact that the model fulfills the large energy limit was not obvious from the start. This is a further test of consistency of the model. Concerning the scaling properties of $\zeta_{||}$ and ζ_{\perp} , the asymptotic E -dependence is not predicted by the large energy limit. As $E \sim M$ at $q^2 = 0$ the Feynman mechanism contribution to the form factors would indicate a $1/E^2$ behaviour rather than the $1/E$ found in the model. Note however that the E -dependence is not rigorously established in QCD.

3 Conclusions

From an effective Lagrangian at the level of mesons and constituent quarks, it is possible to compute meson transition amplitudes by evaluating loops of heavy and light quarks. The agreement with data, when available, is good. The model is able to describe a number of essential features of heavy meson physics in a simple and compact way, in particular Isgur-Wise scaling in the heavy-to-heavy semileptonic decays and the large energy limit for the heavy-to-light ones.

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